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# Advanced Prediction Models

# Beyond Prediction

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- Recall from the introductory class
  - We discussed complex prediction problems and addressed them using
    - Deep learning architectures
    - Graphical models
  - We also discussed complex decisions, especially in the presence of feedback
- A way to make data-driven decisions: we will look at
  - Online machine learning (this lecture)
  - Reinforcement learning (next)
  - Deep reinforcement learning (next to next)

# Examples of Complex Decisions

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- ▶ Inventory Management
  - ▶ Observations: current inventory levels
  - ▶ Actions: number of units of each item to purchase
  - ▶ Rewards: profit
- ▶ Resource allocation: who to provide customer service to first
- ▶ Routing problems: in management of shipping fleet, which trucks / truckers to assign to which cargo

# Reinforcement Learning: The Next Frontier in Data Science

<https://www.technologyreview.com/s/603501/10-breakthrough-technologies-2017-reinforcement-learning/>

**MIT  
Technology  
Review**

Past Lists+

Topics+

Top Stories

10 Breakthrough Technologies

The List x

Years+

Reversing Paralysis

Self-Driving Trucks

Paying with Your Face

Practical Quantum Computers

The 360-Degree Selfie

Hot Solar Cells

Gene Therapy 2.0

The Cell Atlas

Botnets of Things

Reinforcement Learning

**Reinforcement**

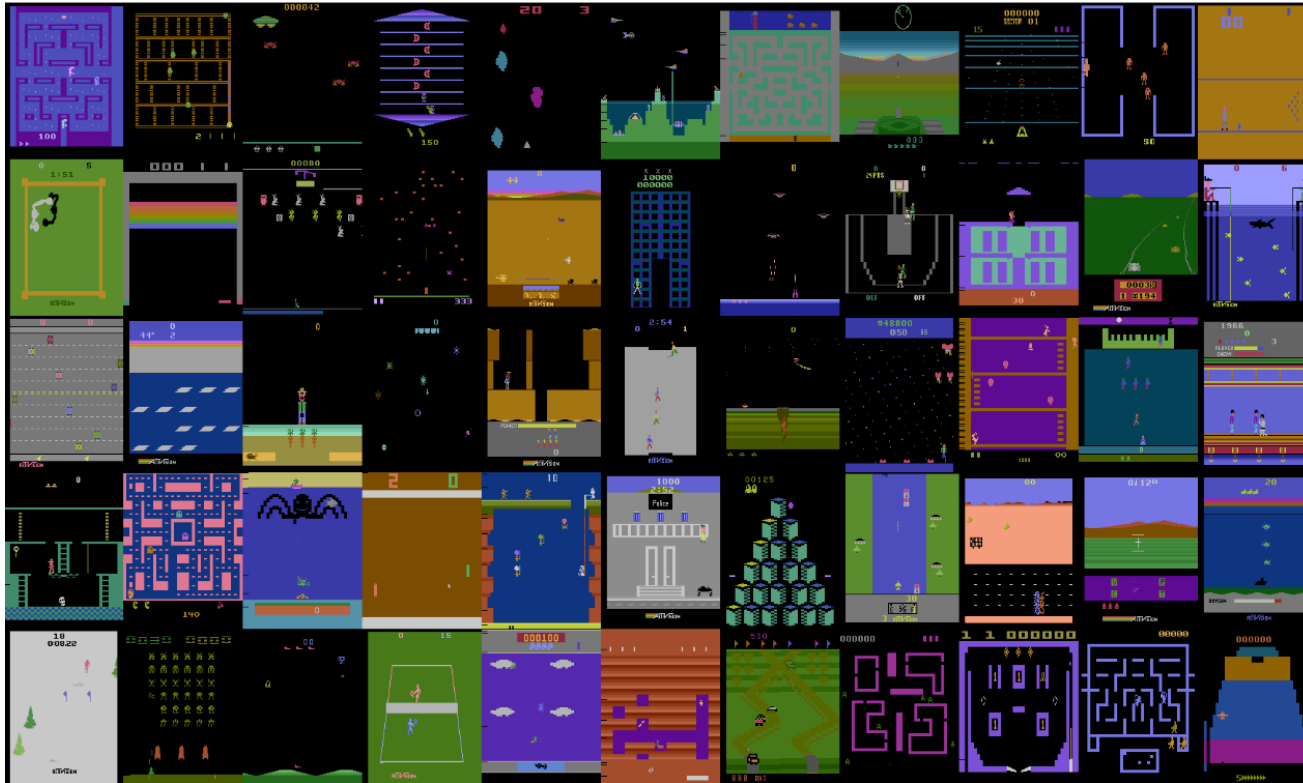
By experimenting  
figuring out how

no programmer could teach them.

March/April 2017 Issue



# Reinforcement Learning: The Next Frontier in Data Science



<sup>1</sup>Figure: Defazio Graepel, Atari Learning Environment

# Reinforcement Learning: The Next Frontier in Data Science



<sup>1</sup>Reference: DeepMind, March 2016

# Today's Outline

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- Online Machine Learning
- A/B Testing
- Multi-armed bandits
- Contextual bandits

---

# Online Machine Learning



# The Gist of Online (Machine) Learning

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1. (Optionally) observe the state of the world (aka **context**)
2. Choose an action
3. Obtain feedback on the chosen action

Repeat

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1. (Optionally) observe the state of the world (aka **context**)
2. Choose an action
3. Obtain feedback on the chosen action

Repeat

**Goal:** Optimize feedback (e.g. maximize reward) for chosen actions

**Assumption:** Agent's actions do not influence future contexts

# MSN Deployment for Personalized News

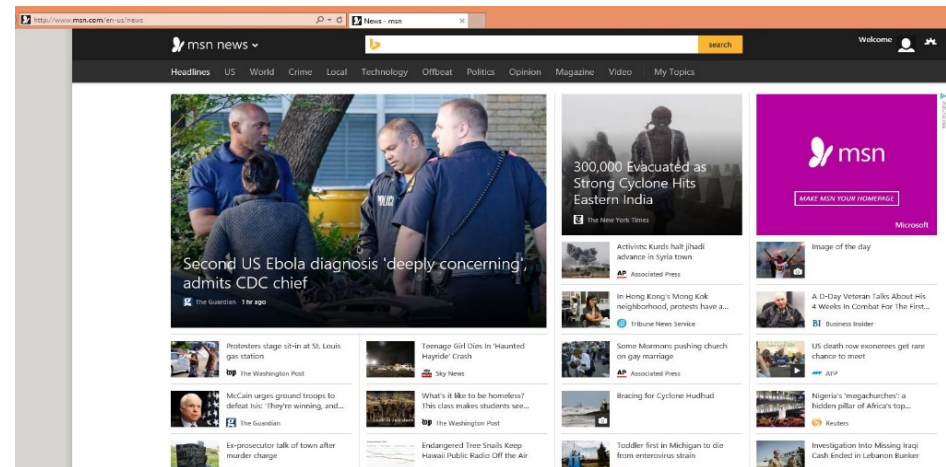
The screenshot displays the MSN homepage with a personalized news feed. At the top, there is a search bar with 'bing web search' and a 'Sign in' button. Below the search bar are navigation links for Outlook.com, Store, Skype, Rewards, Office, OneNote, OneDrive, Maps, and Facebook. A horizontal menu lists various categories: DATING, NEWS, WEATHER, ENTERTAINMENT, SPORTS, MONEY, LIFESTYLE, HEALTH & FITNESS, FOOD & DRINK, TRAVEL, AUTOS, and VIDEO. The main content area features a carousel of video thumbnails with titles like 'Models devour Buffalo wings', 'Sanders talks Trump, Clinton', and 'Stewart returns to 'The Daily Show''. To the right, there are news snippets such as 'Marjorie Lord, 'Danny Thomas Show' star, dies' and '7 year-end retirement mistakes you may want to avoid'. A large advertisement for Toyota Camry is prominent on the right side, featuring a red car and a woman in a Santa hat. Below the main content, there are sections for weather (MONTREAL, CANADA), a 'Daily Deal' for an Asus laptop, and '15 ways to drink coffee that will change your mornings forever'. At the bottom, there are four columns of 'EDITORS' PICKS', 'BEST OF WEEK'S VIDEO', 'CAREERS', and 'WEEKEND READS'.

<sup>1</sup>Reference: Alekh Agarwal et al., <http://arxiv.org/abs/1606.03966>

# MSN Deployment for Personalized News

Loop:

1. User **arrives** at MSN with browsing history, user account, previous visits,...
2. Microsoft **chooses** news stories, ...
3. User **responds** to content (clicks, navigation, etc)

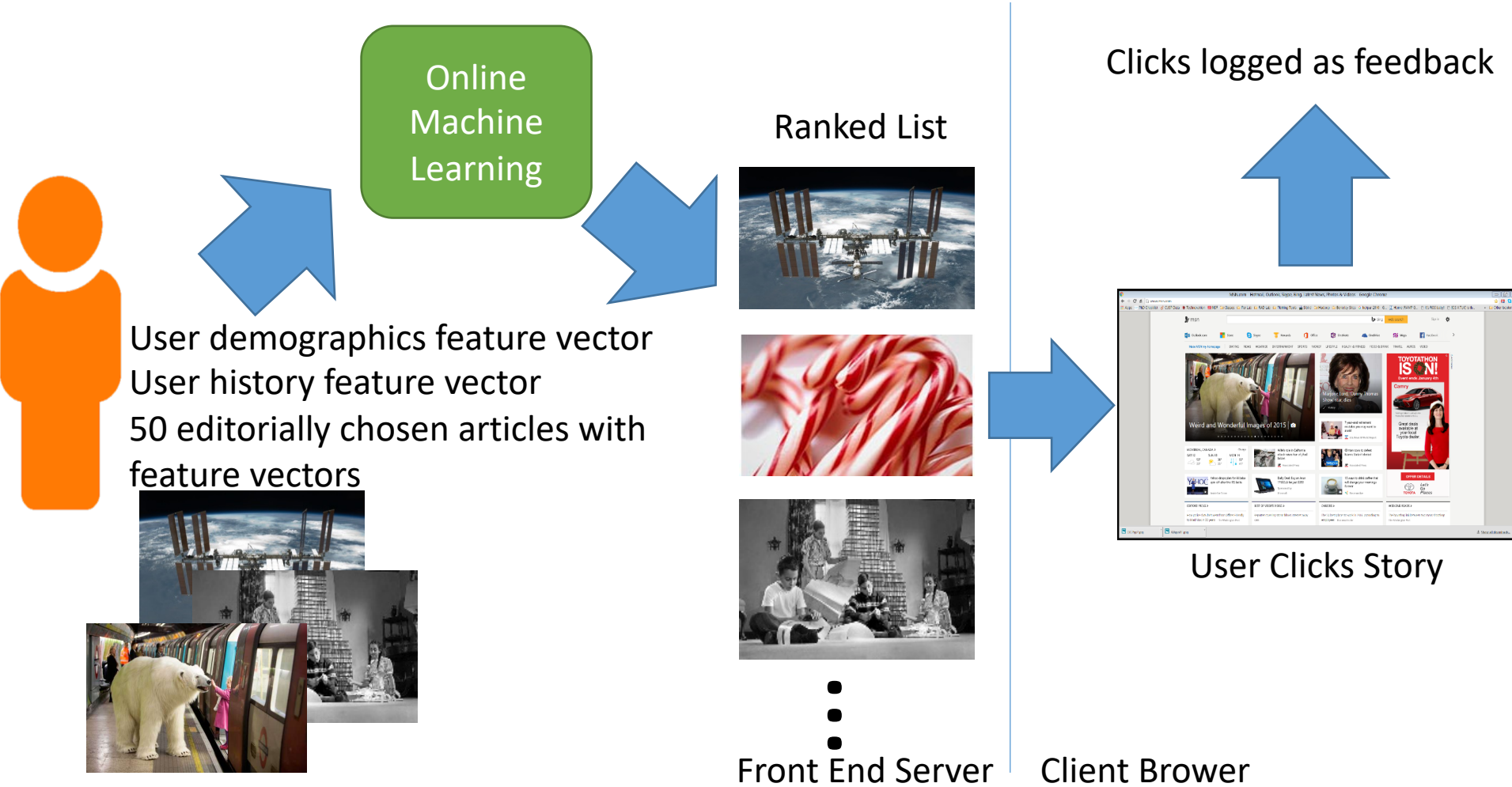


**Goal:** Choose content to yield desired user behavior

**Assumption:** Recommendations to one user do not affect other users

<sup>1</sup>Reference: Alekh Agarwal et al., <http://arxiv.org/abs/1606.03966>

# MSN Deployment for Personalized News



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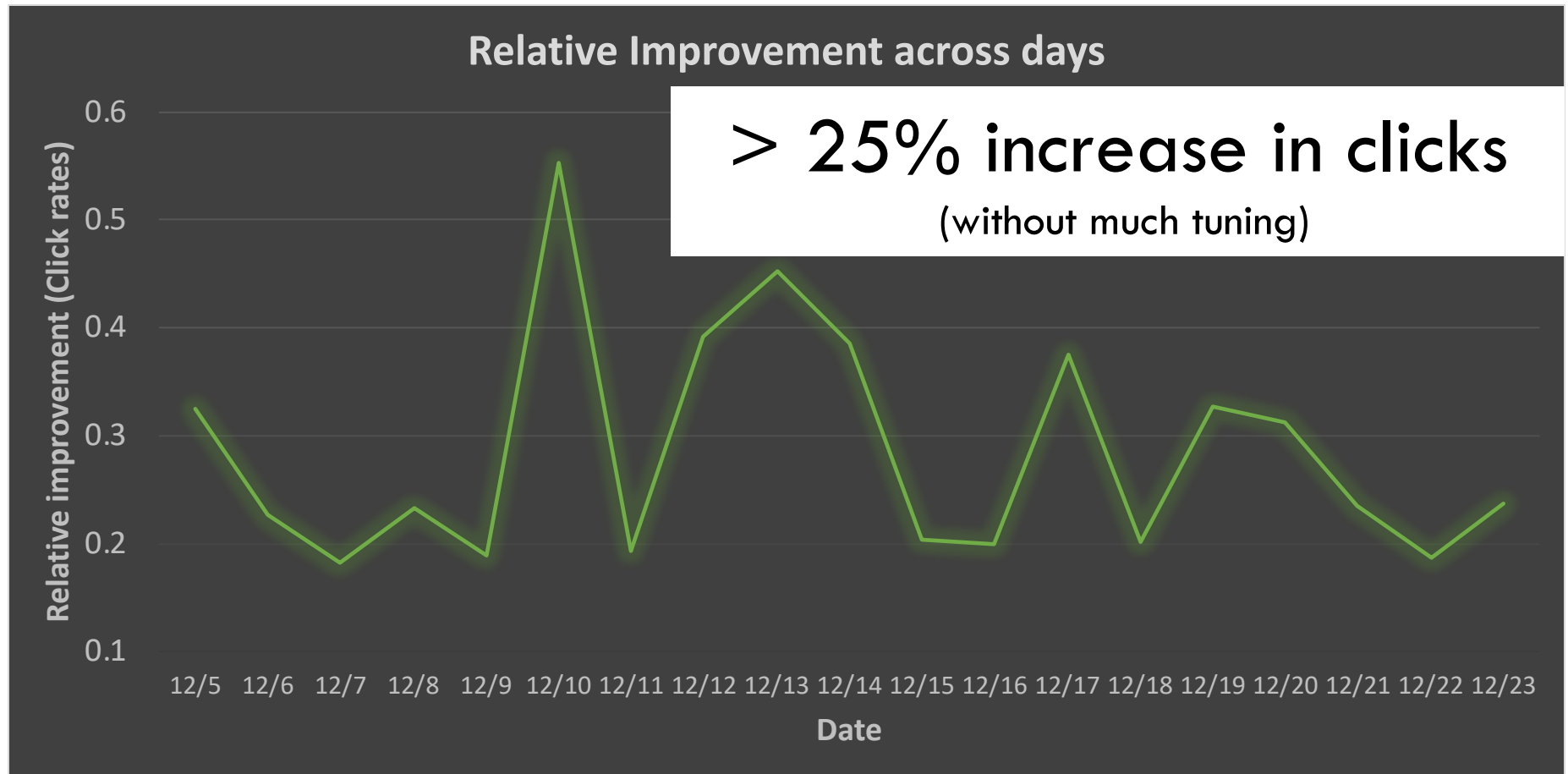
# MSN Deployment for Personalized News

- 10 million+ users
- 1000s of requests per second
- 5% overhead on front end machines
- 10s of servers for training
- 5 minute model update frequency



# MSN Deployment for Personalized News

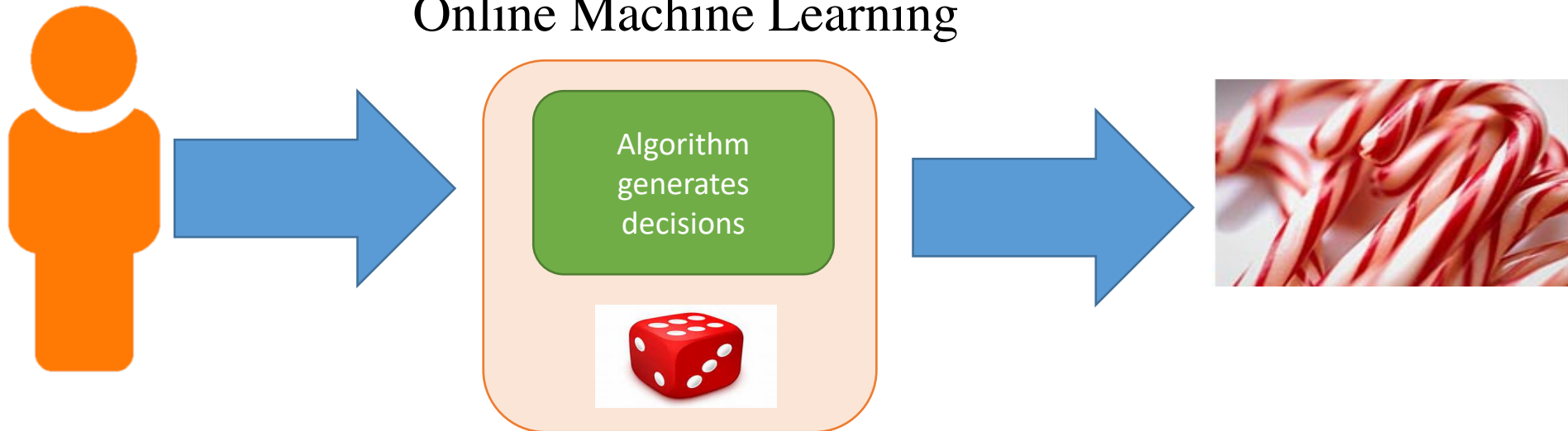
- Relative gains observed



# Multitude of Applications

- Content Recommendation: Apps, Movies, Books, ...
- Personalization of search results
- Customer churn prevention
- Adaptive UI personalization
- ...

## Online Machine Learning





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# Questions?

# Today's Outline

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# A/B Testing

# Motivation for A/B Tests

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- Typical business scenario
  - Say there is a meeting to decide on how to improve a product or service
  - Multiple competing ideas emerge
  - Want to make this decision after making some field observations.
  - How to pick one?
  
- Use A/B testing (this is related to two-sample hypothesis testing)

# Motivation for A/B Tests

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- Full time companies such as Optimizely, Apptimize, APT, Monetate, etc. provide A/B testing services
- Extensively used at
  - Microsoft for Bing.com (see <http://exp-platform.com> )
  - Google, Facebook, Amazon, Airbnb, LinkedIn ...
- Marketing tools
- Clinical trials (\$11b+ market)

# Example with Two Solutions

- Which page has a higher conversion rate?

Doctor FootCare™ Shopping Cart

Home | Products | Learn More | Tips | Testimonials | FAQ | About Us | Contact Us | 1-866-211-9733

Shop With Confidence

- ✓ Satisfaction Guaranteed
- ✓ 30-day, hassle-free Returns
- ✓ 100% Safe, Secured shopping
- ✓ We assure your Privacy

100% Secured Checkout

Item Name	Item Number	Quantity	Remove	Unit Price	Subtotal
Trial Kit	FFCS	1		\$0.00	\$0.00

Update

Total: \$0.00

Select Shipping Method: Standard (\$5.95)

Continue Shopping | Proceed To Checkout

A

Doctor FootCare™ Shopping Cart

Home | Products | Learn More | Tips | Testimonials | FAQ | About Us | Contact Us | 1-866-211-9733

Shop With Confidence

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Trial Kit	FFCS	1		\$0.00	\$0.00

Discount: \$0.00

Total: \$0.00

Enter Coupon Code:

Select Shipping Method: Standard (\$5.95)

Recalculate | Continue Shopping | Proceed To Checkout

B

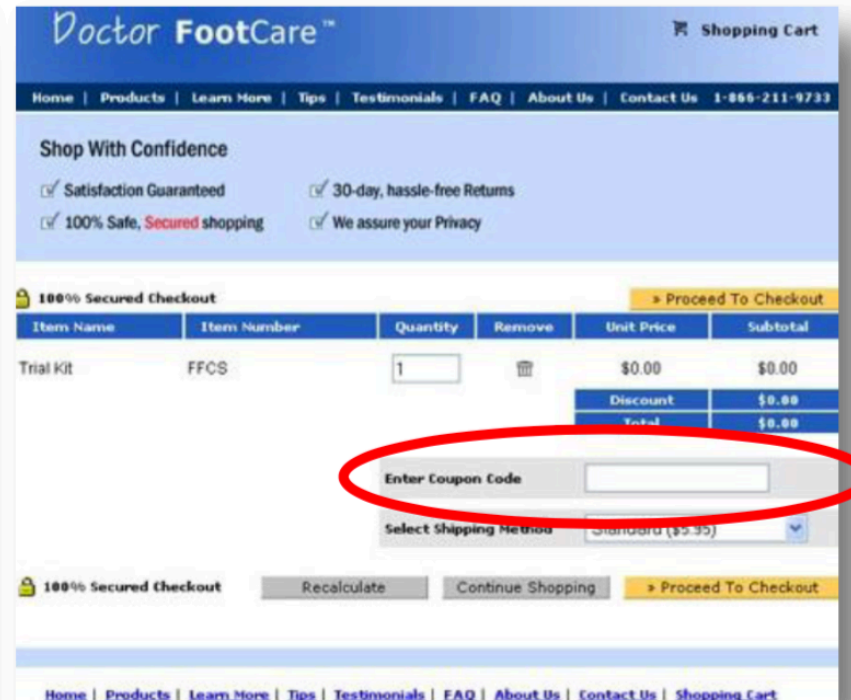
Kumar et al. 2009

# Example with Two Solutions

- Which page has a higher conversion rate?



A



B

Kumar et al. 2009

- With B, site lost 90% of revenue: users want to find coupons to reduce price

# A/B Testing Setup

---

- First we will ignore the online aspect of the problem
- That is, we will ignore instantaneous feedback
  
- We will only use these feedbacks at the end of a period
- In particular,
  - They will be used to decide on good recommendation policies



# A/B Testing Setup

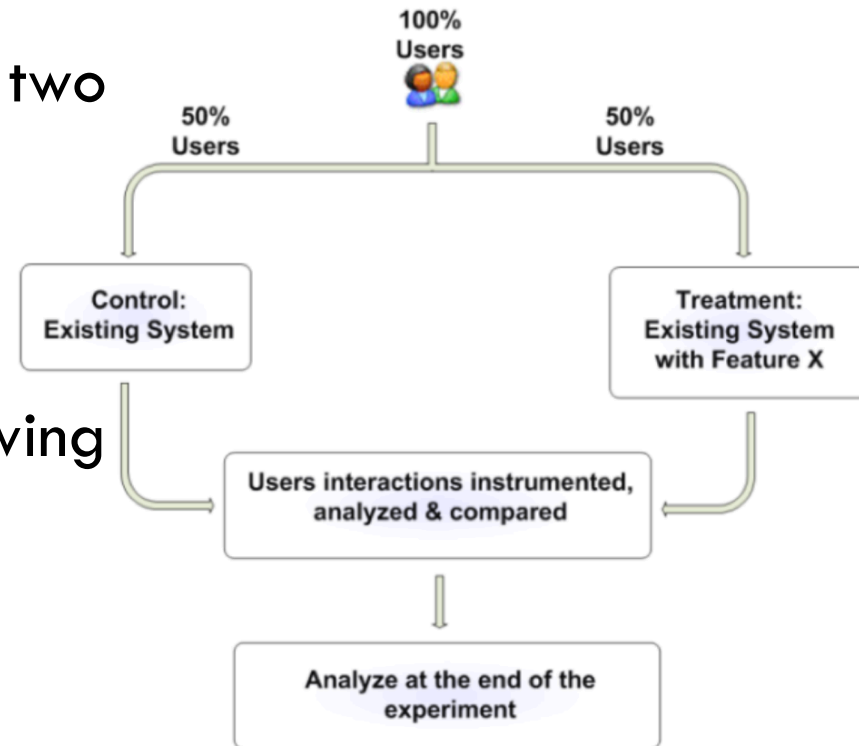
- A/B testing is about showing users two solutions



- And figuring out if solution A is different than solution B

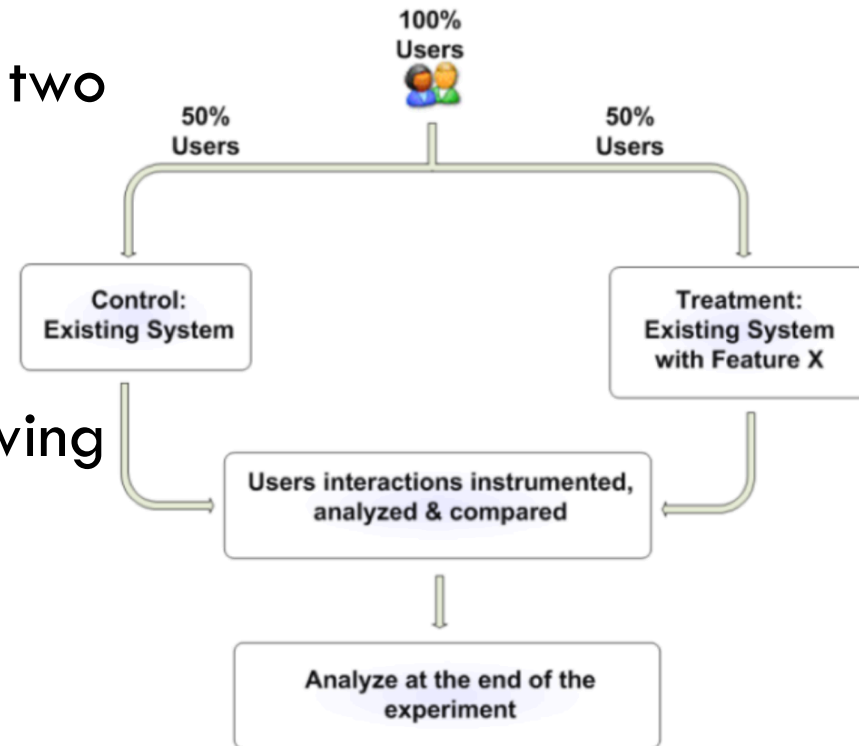
# A/B Testing Setup

- A/B testing is about showing users two solutions
  - A (control)
  - B (treatment)
- Randomly split the users while showing



# A/B Testing Setup

- A/B testing is about showing users two solutions
  - A (control)
  - B (treatment)
- Randomly split the users while showing
- Collect the outcomes and decide which option was better
  - Best scientific way to establish cause-effect relationship
  - Compared to offline data analysis (error prone)



# A/B Testing is Two Sample Testing

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- A/B testing is about collecting statistics across two groups
- Randomized assignment of the two solutions to each user is a key requirement
  - Eliminates biases and confounding
- Say each group of users has true mean effect  $\mu_1$  and  $\mu_2$
- From data, we want to infer whether
  - These are different (statistical significance)?
  - Same?
  - Which is larger?

# Types of Hypothesis Tests

- Fisher
  - Reject  $H_0$  (no acceptance as such)
  - More data typically leads to rejection
- Neyman-Pearson
  - Compare  $H_0$  to  $H_1$
  - Find likelihood ratio  $P(Data|H_0)/P(Data|H_1)$
- Bayesian
  - Compute  $P(H_0|Data)/P(H_1|Data)$
  - Similar to Neyman-Pearson when  $P(H_0) = P(H_1)$

# A/B Testing Pros

---

- Very intuitive setup and conclusions
- Field experiment decides the worth of a feature/offering, not gut instinct
- Most used in industry! (compared to bandit techniques)
  - Also called split or bucket testing
- Need not be a one time process
  - Can repeat if you think users have changed in terms of their preferences

# A/B Testing Cons

---

- Has many bells and whistles to make it work
  - Especially because most treatment effects show small incremental improvement
  - See <http://exp-platform.com> for an extensive list of issues that affect A/B testing
- What if we can change who sees what treatment (action) dynamically?
  - Leads to Multi-Armed Bandit problems.
- What if we want to **optimize** over several options dynamically depending on context?
  - Leads to Contextual Bandit problems.

---

# Questions?



# Today's Outline

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- Online Machine Learning
- A/B Testing
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---

# Bandit Problems

# The Multi-armed Bandit Problem

- Multi-armed bandit (MAB) problem involves the following in each interaction



- pulling an arm = making a choice (which ad/color to display)
- reward/regret = measure of success (user-click, item-buy)

# The Formal Setting

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## Problem Formulation

Consider  $K$  arms (actions) each correspond to an unknown distribution  $\{\nu_k\}_{k=1}^K$  with values bounded in  $[0, 1]$ .

- At each time  $t$ , the agent pulls an arm  $I_t \in \{1, \dots, K\}$  and observes a reward  $x_t \sim \nu_{I_t}$  (i.i.d. sample from  $\nu_{I_t}$ ).
- The objective is to maximize the expected sum of rewards.

## Notations

- mean of each arm:

$$\mu_k = \mathbb{E}_{X \sim \nu_k}[X]$$

- mean of the best arm:

$$\mu^* = \max_k \mu_k$$

# MAB Performance

- It is an online problem.
- We need to come up with algorithms/strategies.
  - Example:
    - a round-robin strategy
    - A constant strategy (bad idea!)

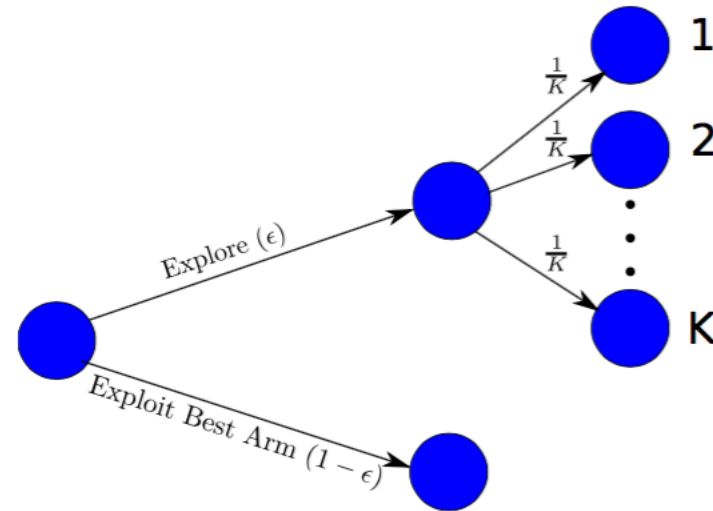
To evaluate the performance of a strategy

**Cumulative Regret**

$$R_n = n\mu^* - \sum_{t=1}^n x_t$$

**Objective:** find a strategy with small *expected cumulative regret*  $\mathbb{E}[R_n]$

# The Epsilon-Greedy Algorithm



Strategy =  $\epsilon \cdot \text{Scientist} + (1 - \epsilon) \cdot \text{Businessman}$

At each time  $t$

- With probability  $1 - \epsilon$ , pick the subjectively best arm
- With probability  $\frac{\epsilon}{K}$ , pick a random arm

# The Epsilon-Greedy Algorithm Intuition

- How can we do well? We need to explore the arms. We also need to exploit what we have learned so far.

## Scientist View

- Explore new ideas



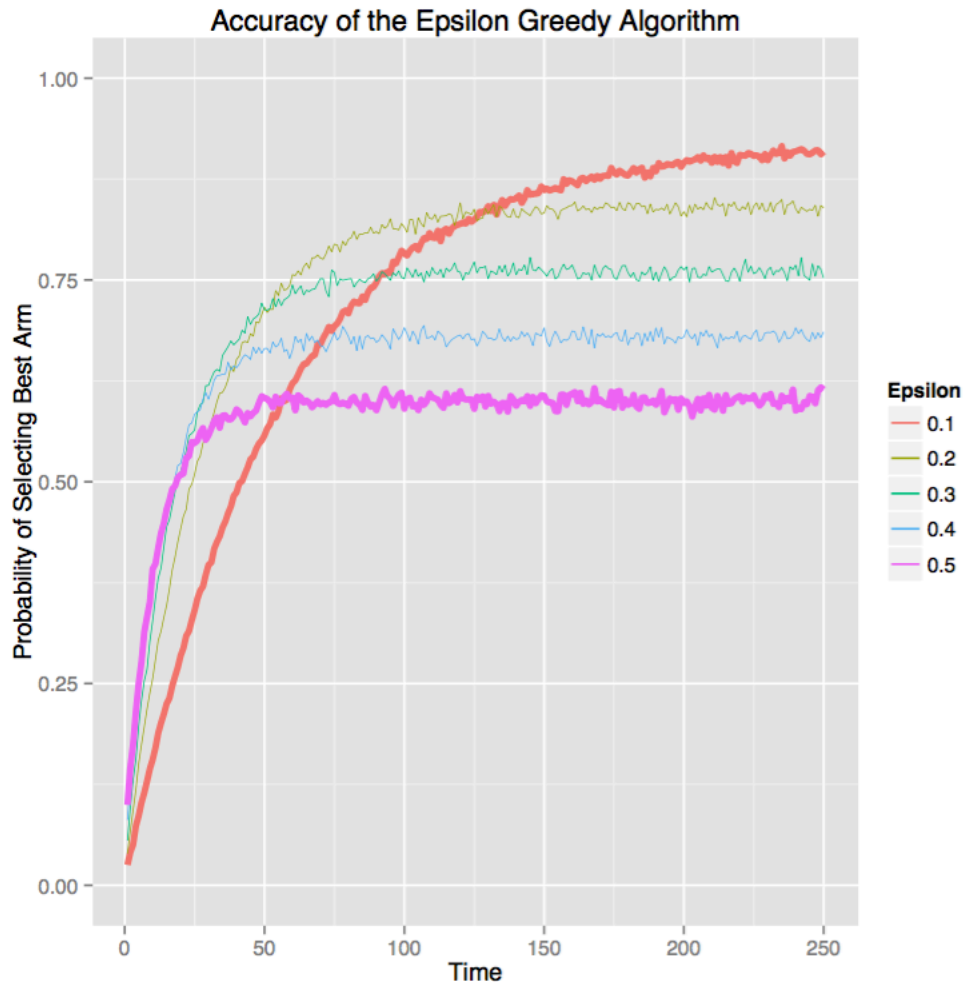
## Businessman View

- Exploit best idea found so far



# Epsilon-Greedy Synthetic Experiment

5 Bernoulli arms with reward probabilities **0.1, 0.1, 0.1, 0.1, 0.9**



$\epsilon = 0.1$  (Businessman)

- Learns slowly
- Does well at the end

$\epsilon = 0.5$  (Scientist)

- Learns quickly
- Doesn't exploit at the end



# The Upper Confidence Bound (UCB) Algorithm

- Lets look at a slightly more involved algorithm: UCB

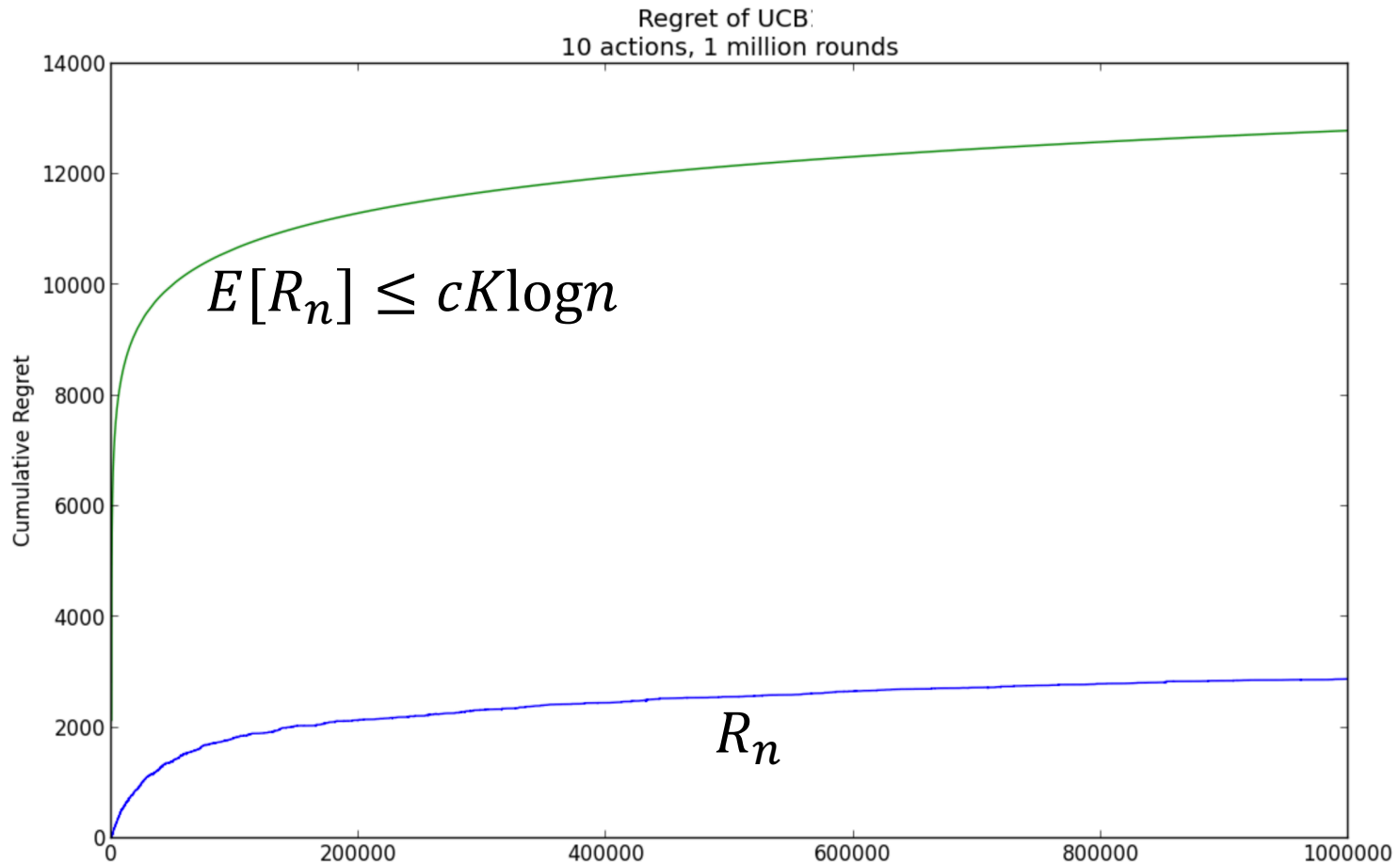
Upper confidence bound (UCB) strategy selects an arm at time  $t$  that

$$I_t = \arg \max_k B_{t, T_k(t-1)}(k) \quad , \quad B_{t,s}(k) = \hat{\mu}_{k,s} + \sqrt{\frac{2 \log t}{s}}$$

$\hat{\mu}_{k,s} = \frac{1}{s} \sum_{i=1}^s x_{k,i}$  is the **empirical mean** of arm  $k$  at time  $s$

# UCB Synthetic Experiment

- 10 actions,  $10^6$  interactions (is this realistic?)
- Reward for each action has mean  $0.5/k$  ( $5 \leq k \leq 15$ )



# The Thompson Sampling Algorithm

- A Bayesian algorithm for MAB problems is as follows

In Thompson [1933] the following strategy was proposed for the case of Bernoulli distributions:

- Assume a **uniform prior** on the parameters  $\mu_i \in [0, 1]$ .
- Let  $\pi_{i,t}$  be the **posterior distribution** for  $\mu_i$  at the  $t^{\text{th}}$  round.
- Let  $\theta_{i,t} \sim \pi_{i,t}$  (independently from the past given  $\pi_{i,t}$ ).
- $I_t \in \operatorname{argmax}_{j=1,\dots,K} \theta_{j,t}$ .

# Thompson Sampling: Conjugate Priors

A family of prior distribution

$$\mathcal{P}_A = \{p_\alpha(\theta) \mid \alpha \in A\}$$

is said to be **conjugate** to a model  $\mathcal{P}_\Theta$ , if, for a sample

$$X^{(1)}, \dots, X^{(n)} \stackrel{\text{i.i.d.}}{\sim} p_\theta \quad \text{with} \quad p_\theta \in \mathcal{P}_\Theta,$$

the distribution  $q$  defined by

$$q(\theta) = p(\theta \mid x^{(1)}, \dots, x^{(n)}) = \frac{p_\alpha(\theta) \prod_i p_\theta(x^{(i)})}{\int p_\alpha(\theta) \prod_i p_\theta(x^{(i)}) d\theta}$$

is such that

$$q \in \mathcal{P}_A.$$

# Thompson Sampling: Conjugate Priors

---

We say that  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_K)$  follows the Dirichlet distribution and note

$$\boldsymbol{\theta} \sim \text{Dir}(\boldsymbol{\alpha})$$

for  $\boldsymbol{\theta}$  in the simplex  $\Delta_K = \{\mathbf{u} \in \mathbb{R}_+^K \mid \sum_{k=1}^K u_k = 1\}$  and

# Thompson Sampling: Conjugate Priors

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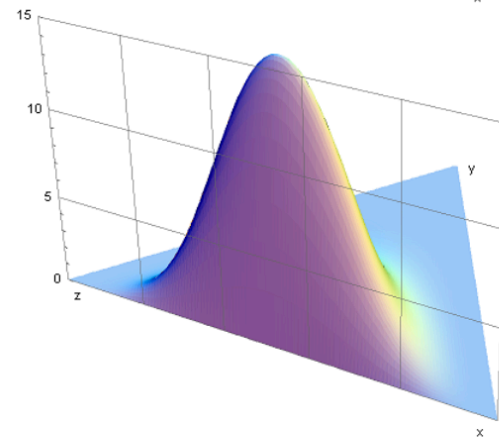
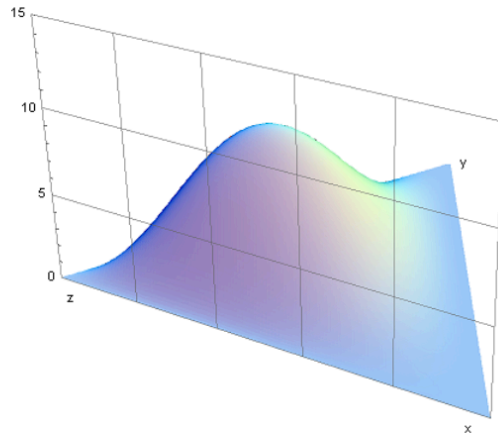
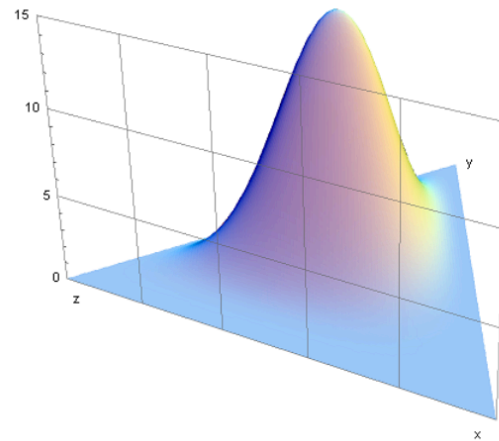
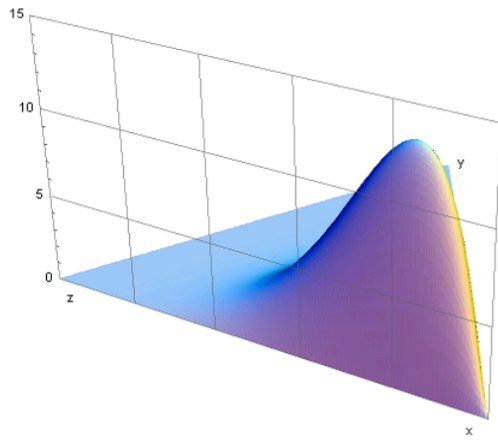
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for  $\boldsymbol{\theta}$  in the simplex  $\Delta_K = \{\mathbf{u} \in \mathbb{R}_+^K \mid \sum_{k=1}^K u_k = 1\}$  and admitting the density

$$p(\boldsymbol{\theta}; \boldsymbol{\alpha}) = \frac{\Gamma(\alpha_0)}{\prod_k \Gamma(\alpha_k)} \theta_1^{\alpha_1-1} \dots \theta_K^{\alpha_K-1}$$

$$\alpha_0 = \sum_k \alpha_k \quad \text{and} \quad \Gamma(x) := \int_0^\infty t^{x-1} e^{-t} dt$$

# Thompson Sampling: Conjugate Priors



# Thompson Sampling: Categorical-Dirichlet Conjugacy

Consider the simple Bayesian Dirichlet-Multinomial model with

- A Dirichlet prior on the parameter of the multinomial:  $\boldsymbol{\theta} \sim \text{Dir}(\boldsymbol{\alpha})$
- A multinomial random variable  $\mathbf{z} \sim \mathcal{M}(1, \boldsymbol{\theta})$

$$p(\boldsymbol{\theta}) \propto \prod_{k=1}^K \theta_k^{\alpha_k - 1} \quad \text{and} \quad p(\mathbf{z}|\boldsymbol{\theta}) = \prod_{k=1}^K \theta_k^{z_k}$$



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Let  $\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(N)}$  be an i.i.d. sample distributed like  $\mathbf{z}$ .

We have

$$p(\boldsymbol{\theta}|\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(N)}) = \frac{p(\boldsymbol{\theta}) \prod_n p(\mathbf{z}^{(n)}|\boldsymbol{\theta})}{p(\mathbf{z}^{(1)}, \dots, \mathbf{z}^{(N)})}$$

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So that  $(\boldsymbol{\theta}|\mathbf{Z}) \sim \text{Dir}((\alpha_1 + N_1, \dots, \alpha_K + N_K))$  with  $N_k = \sum_n z_{nk}$  50

# Non-Probabilistic Setting

---

- Why do we need to assume that the rewards are i.i.d.?
- Can we drop the stochastic assumptions on the rewards?
- Reason #1: These rewards may be the output of a complex process
- Reason #2: These rewards may be generated by an ‘adversary’ (someone who is not random)

# Non-Probabilistic Setting

---

- We can in fact drop the probabilistic reward assumption!
- Template
  - Adversary selects rewards  $x_t(1), \dots, x_t(K)$ , which are not known to the player (us)
  - Player selects arm  $I_t$
  - In full information, player sees  $x_t(1), \dots, x_t(K)$
  - In bandit information setup, player only sees  $x_t(I_t)$

# Exp3 Algorithm

---

Initialization:  $w_1(k) = 1$  for all  $k = 1, \dots, K$

# Exp3 Algorithm

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At each time  $t = 1, \dots, n$ : the player selects an arm  $I_t \sim p_t$ , where

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# Exp3 Algorithm

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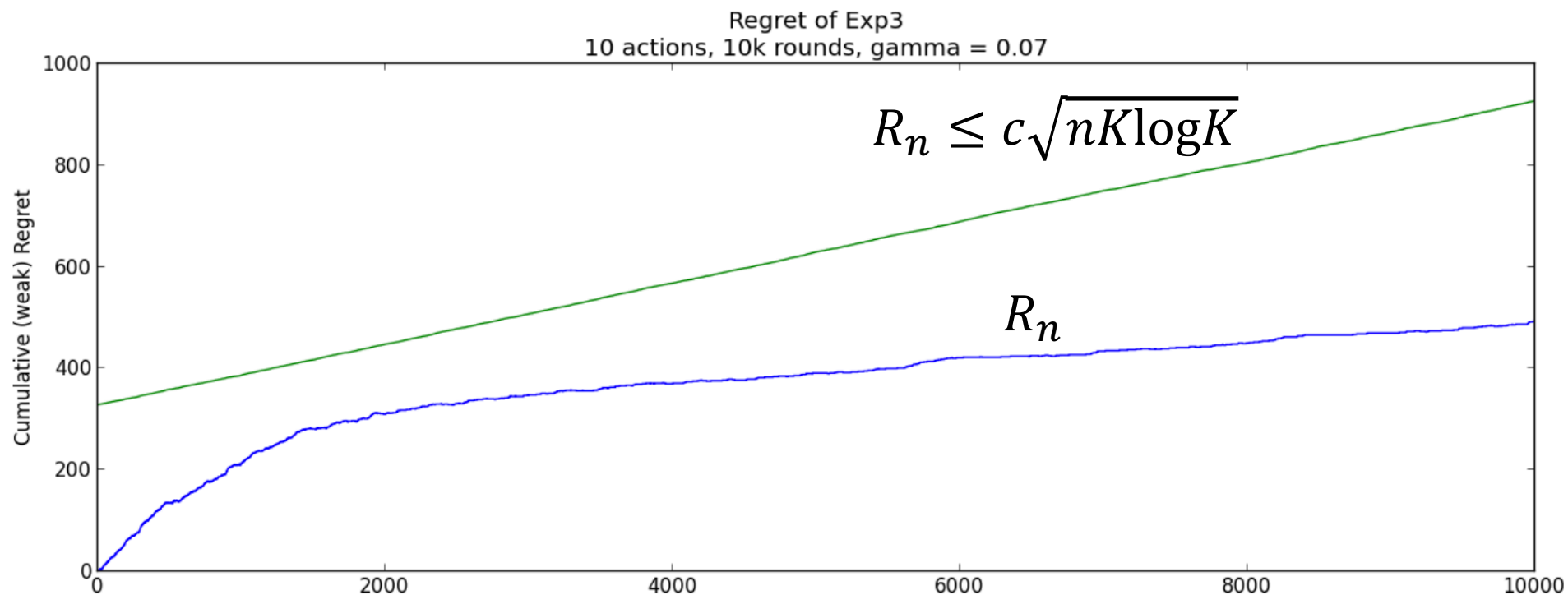
$$p_t(k) = (1 - \gamma) \frac{w_t(k)}{\sum_{i=1}^K w_t(i)} + \frac{\gamma}{K}$$

with  $w_t(k) = e^{\eta \sum_{s=1}^{t-1} \tilde{x}_s(k)}$ , where  $\tilde{x}_s(k) = \frac{x_s(k)}{p_s(k)} \mathbf{1}\{I_s = k\}$ .

$\eta > 0$  and  $\gamma > 0$  are the parameters of the algorithm.

# Exp3 Synthetic Experiment

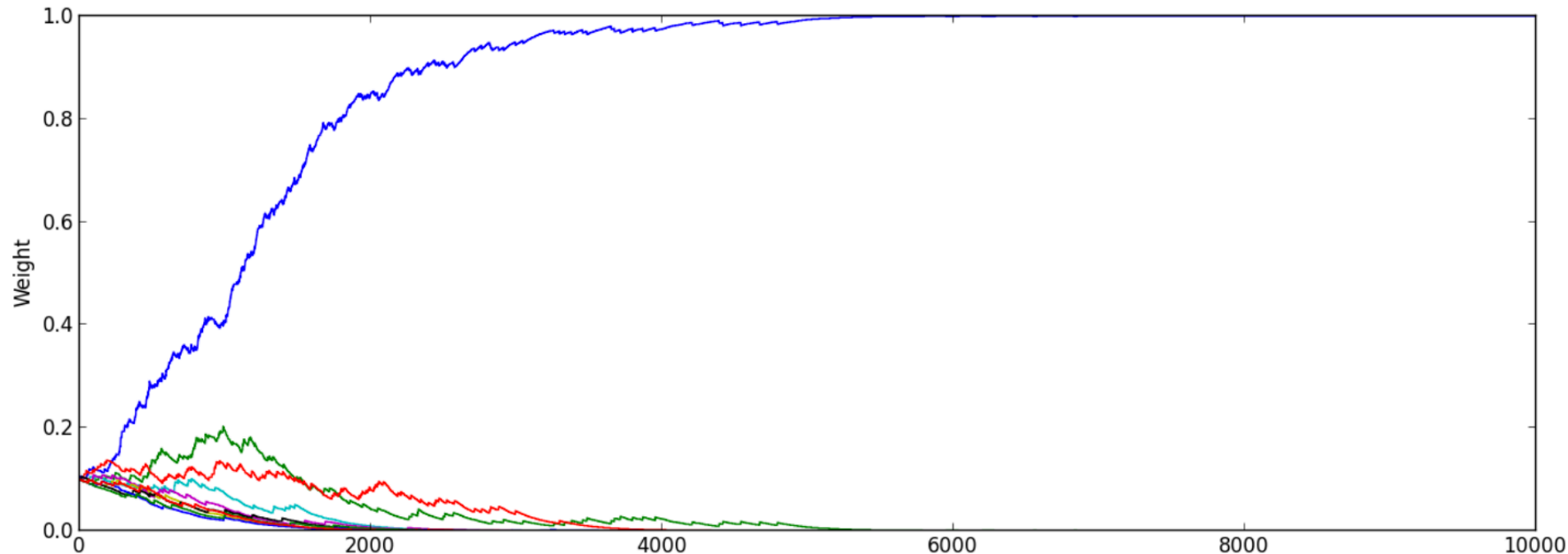
- 10 actions,  $10^3$  interactions
- Reward for each action is Bernoulli with means  $1/k$  ( $2 \leq k < 12$ )





# Exp3 Synthetic Experiment

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# Questions?

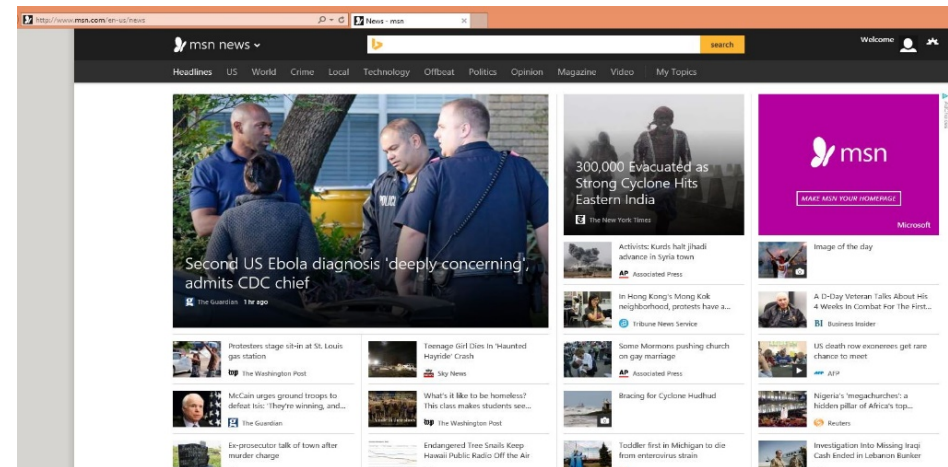
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# Bandits with Contexts

# Recall: MSN Deployment for Personalized News

Loop:

1. User **arrives** at MSN with browsing history, user account, previous visits,...
2. Microsoft **chooses** news stories, ...
3. User **responds** to content (clicks, navigation, etc)



**Goal:** Choose content to yield desired user behavior

**Assumption:** Recommendations to one user do not affect other users

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# Previous Bandit Models are not Enough

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- No context!
- No-carry over effect from one interaction to the next
  - Say users can change behavior by seeing recommendations
  - Can be captured by Reinforcement Learning

# The Contextual Bandit Problem

---

- In the Contextual Bandit problem,
  - Every round, we get context
  - We want to find the best policy (what to do in each context)
  - May not see the same context twice!
- Different from MAB setting because in MAB problems
  - No context
  - We were finding a single best action

# Benefit of Context

- Say we have 5 ads

$a_1 =$  “buy pet lizards”

$a_2 =$  “1-800-petunias”

$a_3 =$  “cheap mp3 players”

$a_4 =$  “find local florists”

$a_5 =$  “affordable dragon souls”

- Say we have 4 policies

- These map context to ads

- Now, lets look at one round of Exp3

- For Exp3, it is as if it has 4 “arms” (one per policy)

# Benefit of Context

- In round  $t$  say the policies recommend the following:

$e_1$  chose  $a_2$

$a_1 =$  “buy pet lizards”

$e_2$  chose  $a_2$

$a_2 =$  “1-800-petunias”

$e_3$  chose  $a_4$

$a_3 =$  “cheap mp3 players”

$e_4$  chose  $a_4$

$a_4 =$  “find local florists”

$a_5 =$  “affordable dragon souls”

- Say Exp3 chose “arm”  $e_1$  by sampling from weights
- And, say  $e_1$ 's ad choice  $a_2$  was clicked



# Benefit of Context

- Exp3 assigns reward  $\tilde{x}_s(e_1) = \frac{x_s(e_1)}{p_s(e_1)}$
- Rest of the arms all get reward 0
- Can we do better?
  - Yes!  $e_2$  also was recommending  $a_2$
  - We should better estimate reward of  $e_2$

$e_1$  chose  $a_2$

$e_2$  chose  $a_2$

$e_3$  chose  $a_4$

$e_4$  chose  $a_4$

# Exp4 Algorithm

---

Initialization:  $\forall \pi \in \Pi : w_t(\pi) = 1$

For each  $t = 1, 2, \dots$ :

1. Observe  $x_t$  and let for  $a = 1, \dots, K$

$$p_t(a) = \frac{\sum_{\pi} \mathbf{1}[\pi(x_t) = a] w_t(\pi)}{\sum_{\pi} w_t(\pi)}$$

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$$p_t(a) = (1 - K\rho_{\min}) \frac{\sum_{\pi} \mathbf{1}[\pi(x_t) = a] w_t(\pi)}{\sum_{\pi} w_t(\pi)} + \rho_{\min},$$

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3. Update for each  $\pi \in \Pi$

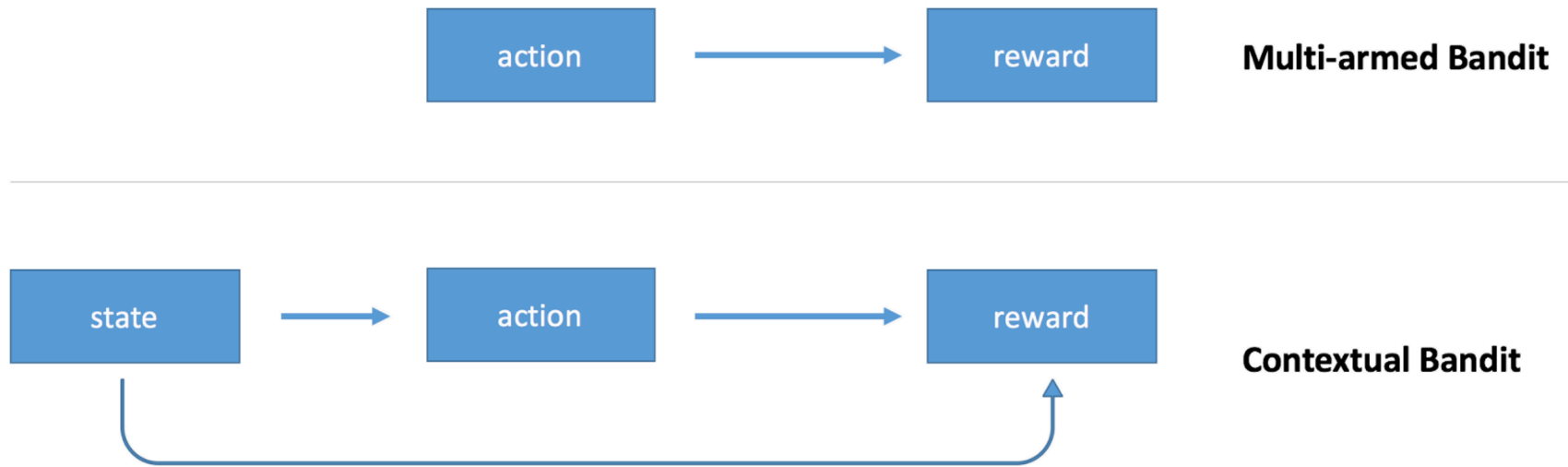
$$w_{t+1}(\pi) = \begin{cases} w_t(\pi) \exp\left(\rho_{\min} \frac{r_t(a_t)}{p_t(a_t)}\right) & \text{if } \pi(x_t) = a_t \\ w_t(\pi) & \text{otherwise} \end{cases}$$

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# Questions?



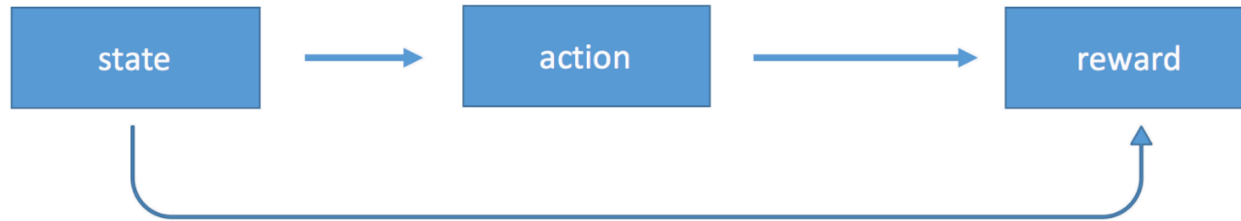
# Reinforcement Learning: Because Contextual Bandit Formulation is not Enough



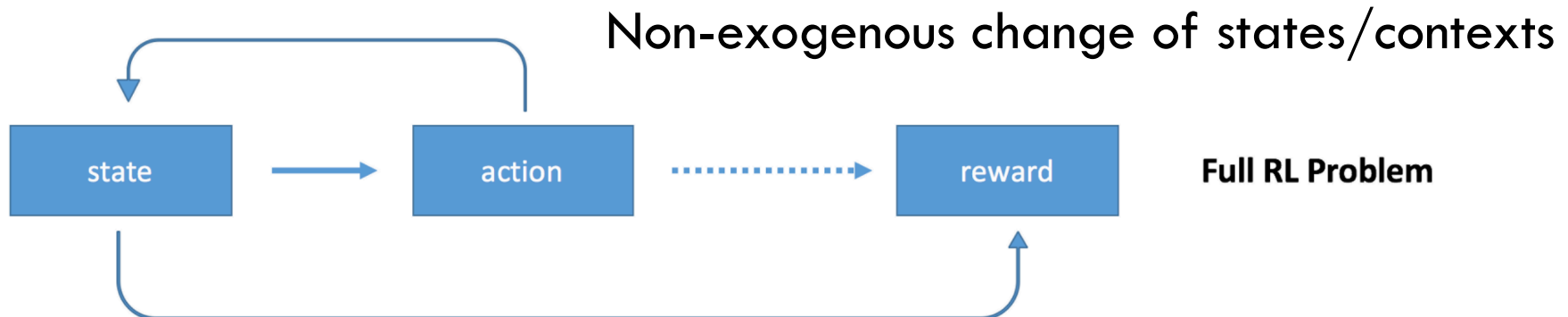
# Reinforcement Learning: Because Contextual Bandit Formulation is not Enough



**Multi-armed Bandit**



**Contextual Bandit**



**Full RL Problem**

# Summary

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- We looked at A/B testing as a way to introduce enhancements in a business product/service
  - May need a lot of examples
  - Is based on the idea of randomized control trials
- We also looked at two new online ML problems
  - Multi-Armed Bandits
  - Contextual Bandits
- Contextual bandits are a special case of reinforcement learning, which we will study next time.

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# Appendix

# Sample Exam Questions

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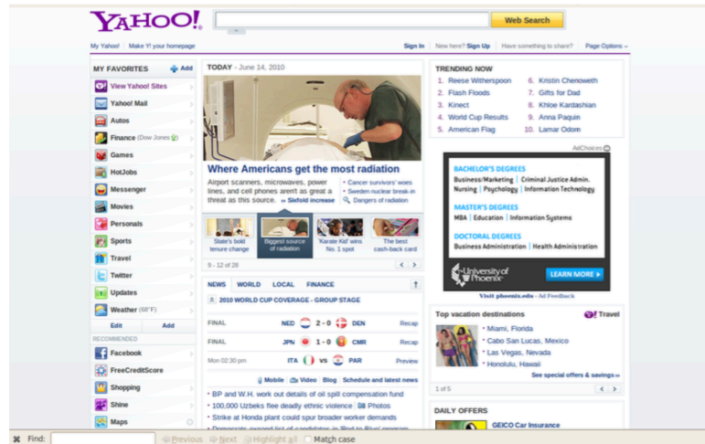
- What is the difference between A/B testing and Multi-armed bandits?
- Can we do A/B testing when we have more than two options?
- What is the role of exploration in the Bandit problems?
- Can Exp3 be used in a stochastic setting?
- How does the contextual problem differ from the non-contextual problem?

# Online ML is Difficult to Deploy

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- Separate teams for each part of the process
- Faulty logging
  - Logging just choice, not probabilities
  - Features not logged and change in time
- Runtime behavior incompatible with the ML
  - Business logic overriding randomization
  - Using the probability as feature for downstream ML
- Subtle errors that are difficult to find in complex systems!

# Contextual Bandit: Website Example



Repeatedly:

1. A user comes to Yahoo! (with history of previous visits, IP address, data related to his Yahoo! account)

# Contextual Bandit: Website Example



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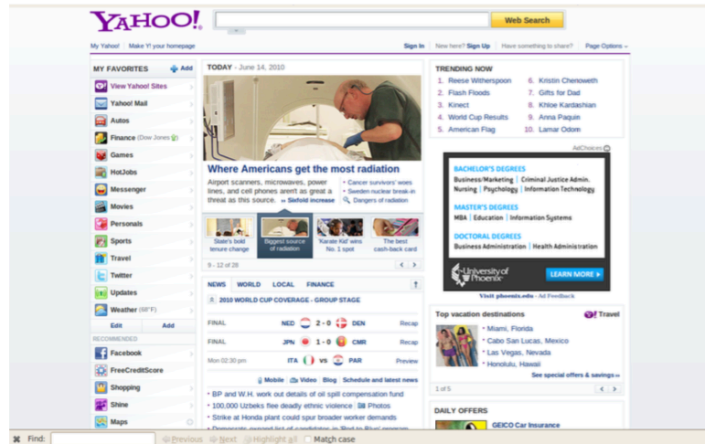
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Yahoo! wants to interactively choose content and use the observed feedback to improve future content choices.

<sup>1</sup>Reference: John Langford (2011)

# Contextual Bandit: Clinical Example



Repeatedly:

1. A patient comes to a doctor with symptoms, medical history, test results
2. The doctor chooses a treatment
3. The patient responds to it

The doctor wants a policy for choosing targeted treatments for individual patients.

# Additional Resources

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- Course at UWash:
  - <http://courses.cs.washington.edu/courses/cse599s/12sp/scribes.html> (lectures 13,14)
- Course at UCSD:
  - <http://cseweb.ucsd.edu/~kamalika/teaching/CSE291W11/> (lecture5)
- Tutorial by Bygelzimer and Langford:
  - [http://hunch.net/~exploration\\_learning/](http://hunch.net/~exploration_learning/)
- Course at UAlberta:
  - <https://sites.ualberta.ca/~szepesva/CMPUT654/>

Note: These are optional. May be slightly theoretical in nature.