X, Y

$X, Y \sim P(X, Y)$

p dim R.V

$P_{XY}(X, Y)$ $\quad P_{XY}(X = x, Y = y)$

Q. $P(Y=y) = \sum_{x} P(X=x, Y=y)$

Q $P(X = (a, c))$

X_1	X_2	Y	
a	c	0	0.1
b	d	0	0.1
a	d	0	0.1
b	c	0	0.1
		1	.6/4
		1	.6/4
		1	.6/4
		1	.6/4

$X, Y \qquad f(X) \approx Y \qquad \underline{(f(X)-Y)^2} \qquad \boxed{|f(X)-Y|}$

$$EPE = E_{XY}\left[\left(\underline{f}(X)-Y\right)^2\right] = \sum_{x,y} \underline{P(X=x, Y=y)}\, \underbrace{(f(x)-y)^2}_{l(x,y,f)}$$

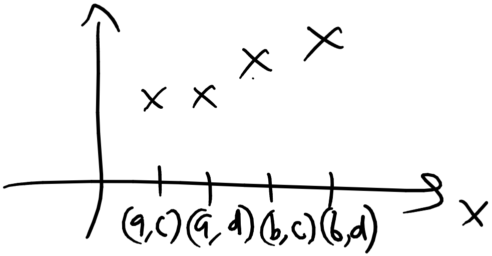
$$E_{XY}\left[l(X,Y,f)\right] = E_X\, E_{Y|X}\left[l(X,Y,f)\,|\,X\right]$$

$$\text{A} \quad P(X=x, Y=y) = P(X=x) \cdot P(Y=y\,|\,X=x)$$

$$EPE(f) = \sum_{x} \sum_{y} P(X=x) \cdot P(Y=y \mid X=x) \cdot (f(x)-y)^2$$

$$= \sum_{x} P(X=x) \left[\sum_{y} P(Y=y \mid X=x) \cdot (f(x)-y)^2 \right]$$

$$g(x) = E_{Y \mid X=x}\left[(f(x)-Y)^2 \right]$$



Solve for each of the 4 values of x

$$\min_{u} E_{Y|X=x}\left[(u-Y)^2 \,\middle|\, X=x \right]$$

$$\frac{d}{du}\left(\sum_{y}\left(P(Y=y|X=x)\cdot(u-y)^2\right)\right)$$

$$= \sum_{y} P(Y=y|X=x)\cdot 2(u-y) = 0$$

$$\underbrace{\sum_{y} P(Y=y|X=x)}_{=1}\cdot u\cdot \cancel{2}$$

$$= \sum_{y}\left(P(Y=y|X=x)\cdot y\right)\cancel{2} = E_{Y|X=x}[Y]$$

the regression function $f^{best}(X) = E_{Y|X}[Y|X]$

$$f^{best}(x) = \underbrace{E[Y|X=x]}_{\approx \frac{1}{10}\sum_{i=1}^{10} y_i}$$

$\approx \frac{1}{10}\sum_{i=1}^{10} y_i$ of those observations for which $\underbrace{x_i = x}$

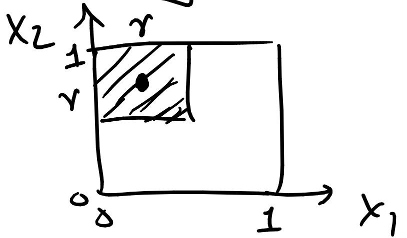
① k-nn(x)

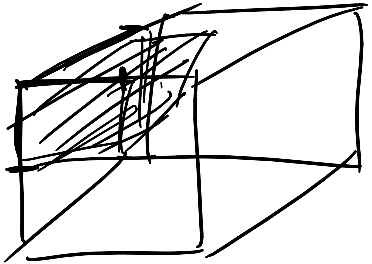
X, Y

$$\begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix}_{N\times p} \quad \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}_{N\times 1}$$

$\star$ $\boxed{x_i \approx x}$

Curse of dimensionality:





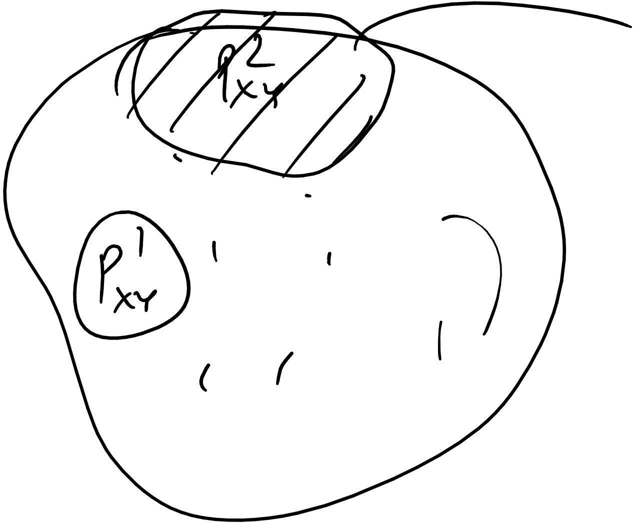
$$\frac{1}{4} = h \cdot w \cdot l$$

$$\frac{1}{4} = r^3$$

$$0.63 = \left(\frac{1}{4}\right)^{1/3} = r$$

$$f^{best}(x) = \underline{E\left[Y \mid X = x\right]}$$

$$f(X) = \beta^T X$$



$$E[Y \mid X = x] = \beta^T x$$

$$X \sim N\left(0, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$Y \mid X \sim N(\beta^T X, 1)$$

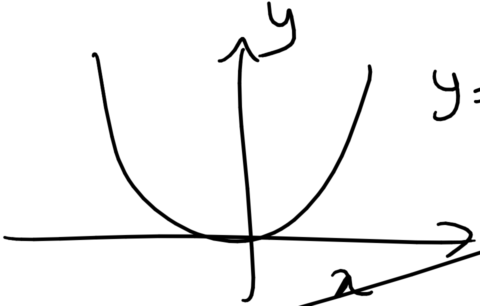
$$Y = \beta^T X + \varepsilon$$

$$N(0, \sigma^2)$$

$$f(X) = \beta^T X \qquad \min_{\beta} EPE(\beta)$$

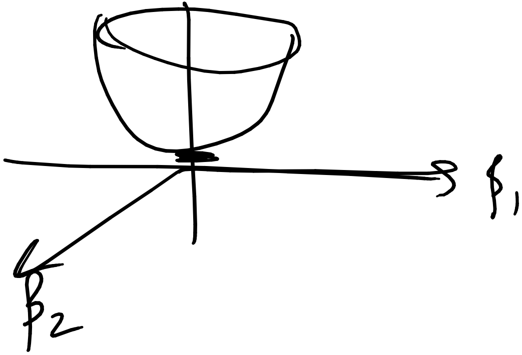
$$E_{XY}\left[(\beta^T X - Y)^2\right]$$

$$\beta = \left(E_X\left[X X^T\right]\right)^{-1} E_{XY}\left[X Y\right]$$



$$y = x^2$$

$$\underbrace{\frac{1}{N}\sum_{i=1}^{N}\left(x_i^T\beta - y_i\right)^2}$$



$$\frac{\partial}{\partial\beta} = 0$$

$$\hat{\beta} = (X^T X)^T X^T Y$$

1

$$\beta^T X = \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_p \cdot X_p + \beta_0 1$$

$$X = \begin{array}{c} \begin{array}{ccc} X_1 & \cdots & X_p \end{array} \\ \begin{bmatrix} \downarrow & & \downarrow \end{bmatrix} \end{array}_{N \times p} \longrightarrow X^{New} = \begin{array}{c} \begin{array}{cccc} X_1^2 & X_1 & \cdots & X_p \end{array} \\ \begin{bmatrix} \vdots & & & \\ & & & \end{bmatrix} \end{array}_{N \times (p+1)}$$

Categorical

$$X_1 = \begin{cases} \text{True} & 0 \\ \text{False} & 1 \\ \text{No Answer} & 2 \end{cases}$$

	X_1	X'_1	X'_2	X'_3
1	True	1	0	0
2	NoAns	0	0	1
3	NoAns	0	0	1
4	True	1	0	0
5	f	0	1	0

EPE for Classification

$$\text{Loss}\left(G, \hat{G}(X)\right)$$

$X, G \qquad p_{XG}$

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ & & 0 & 1 \\ & & & 0 \\ & & & & 0 \end{pmatrix}$$

$K \times K$

$$G \in \{1, \dots K\}$$

$$\mathrm{EPE}\left(\hat{G}(x)\right)$$

$$= E_X \sum_{k=1}^{K} L\left(G=k, \hat{G}(X)\right) \cdot P\left(G=k \mid X\right)$$

$$= E_{XG}\left[L\left(G, \hat{G}(X)\right)\right]$$

$X = x$

$$\underbrace{\sum_{k=1}^{K} L\left(G=k, \hat{G}(x)\right) \cdot \underline{\underline{P\left(G=k \mid X=x\right)}}}$$

$P(G=1 \mid X=x)$

$P(G=2 \mid X=x)$

$\vdots$

$P(G=K \mid X=x)$

$$\underline{\underline{\hat{G}(x) = \underset{k=1,\dots,K}{\arg\max}\ \underline{\underline{P(G=k \mid X=x)}}}}$$

<u>Bayes Classifier</u>

$$L(G=1,\hat{G}(x))\cdot P(G=1|X=x) + L(G=2,\hat{G}(x))\cdot P(G=2|X=x) + L(G=3,\hat{G}(x))\, P(G=3|X=x)$$

0.9   .05   .05

x 1

$K=3$

Q. What should be $\hat{G}(x)$

1, 2, 3 ..
cat dog human.

$$\hat{G}(x)=1 \qquad \text{Loss} = P(G=2|X=x) + P(G=3|X=x)$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad \begin{array}{c} 1 \\ 2 \\ 3 \end{array}\begin{bmatrix} .9 \\ .05 \\ .05 \end{bmatrix}$$

3×3

2, 1, 1, 3