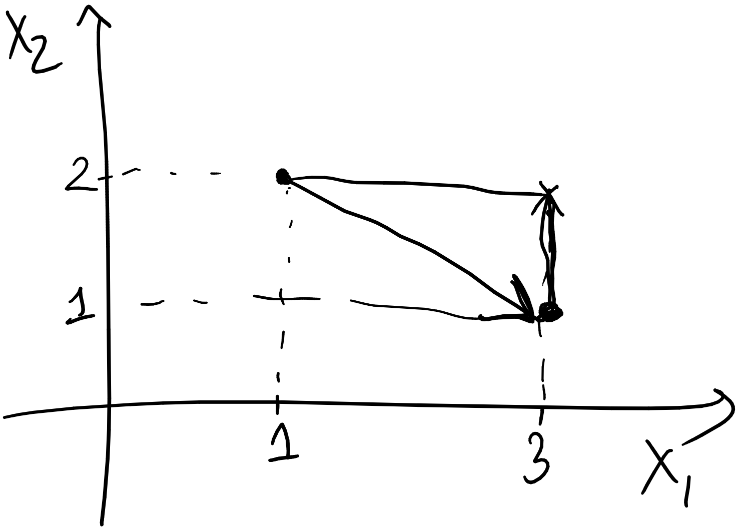


$x^0 = (1, 2)$

$3 \sim P(X_1 \mid X_2 = 2)$

$1 \sim P(X_2 \mid X_1 = 3)$

Time Series data:

$X_{N \times p}$ $\quad Y_{N \times 1}$

$$\begin{matrix} x_1^T \\ x_2^T \\ x_3^T \\ \vdots \\ x_N^T \end{matrix} \quad \longleftarrow \text{Correlation.}$$

$$\begin{matrix} x_i^T & y_i \\ x_j^T & y_j \end{matrix}$$

$$\underbrace{W_1, W_2, \ldots\ldots W_N, \ldots\ldots}_{\text{stochastic process}}$$

MLE

θ

$$P_\theta(W_1, \ldots\ldots W_N)$$

$$\prod_{i=1}^{N} P(W_i)$$

moments of RVs

$$E[W_t], \quad Cov(W_{t+\tau}, W_t)$$

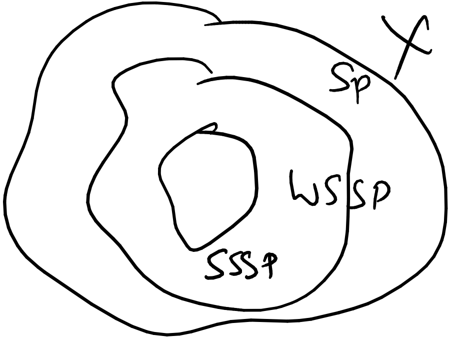
$W_1, \dots\dots W_N$

$\omega_1, \dots\dots \omega_N$ 1 trajectory

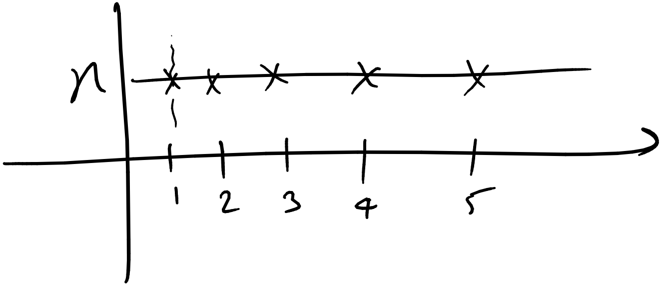
Want: take averages across "time"

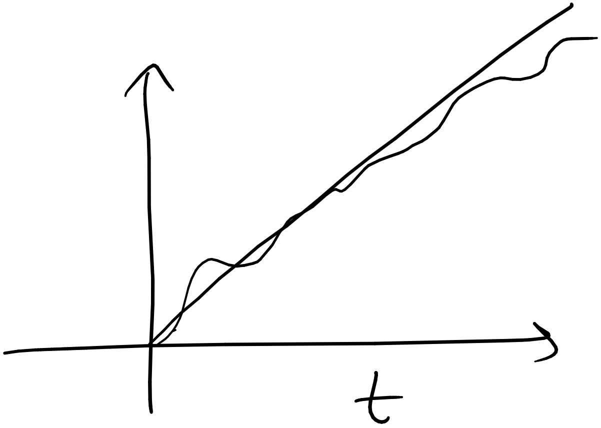
Weakly stationary Stochastic process:

- $E[W_t] = \underset{\uparrow \in \mathbb{R}}{\mu}$ Constant.
- $Cov(W_{t+\tau}, W_t) = \underset{\uparrow \in \mathbb{R}}{\gamma_\tau}$



[Strong stationarity : $P_\theta(W_1, \ldots, W_N) = P_\theta(W_{1+\tau}, \ldots, W_{N+\tau})$]



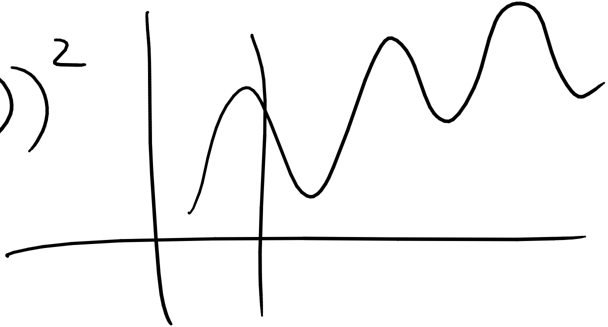


trends.

seasonalities

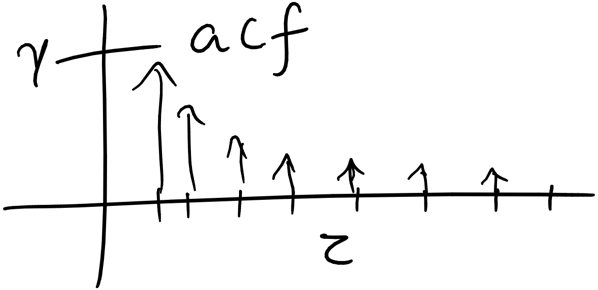
$$W_t \approx \hat{\beta}_0 + \hat{\beta}_1 t$$

$$\min_{\theta_1, \theta_2} \left(W_t - \theta_1 \sin(\theta_2 t)\right)^2$$



$$\mathrm{Cov}(W_{t+\tau}, W_t) = \gamma_\tau$$

$$\tau = 0 : \quad \mathrm{Var}(W_t) = \gamma_0$$



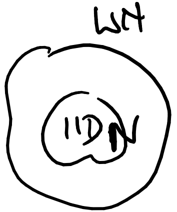
$$\frac{\gamma_\tau}{\gamma_0} \;:\; \text{auto correlation function}$$

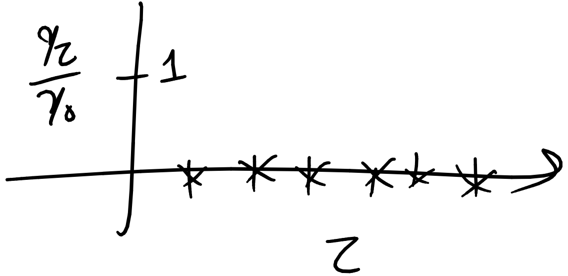
White noise SP.

$$E[W_t] = 0$$

$$E[W_t^2] = \sigma^2$$

$$E[W_t W_{t+\tau}] = 0 \quad \tau \neq 0$$





eg: Non-stationary $\underline{\underline{SP}}$: Random Walk.

$$S_0 = 0$$

$W_1, \ldots W_N$ — WN

$$S_1 = W_1$$

$$S_t = W_1 + W_2 + \cdots + W_t$$

$$Cov(S_t, S_{t+\tau}) = Cov(\underline{W_1 + \cdots W_t}, \underline{W_1 + \cdots + W_t + W_{t+1} + W_{t+2}})$$

$$= E[(W_1 + \cdots + W_t)(\quad)]$$

$$= t\,\sigma^2$$

1st linear model : AR(P)

$\{\varepsilon_t\}_{t=1}^{\infty} \leftarrow$ WN

AR(1) : $W_t = \phi W_{t-1} + \varepsilon_t$

$Var(\varepsilon_t) = \sigma^2$

$W_t = \varepsilon_t + \phi \varepsilon_{t-1} + \phi^2 \varepsilon_{t-2} + \dots + \phi^t W_0$

if $\varepsilon_t \perp W_{t-1}, W_{t-2}, \dots W_0$

then $\{W_t\}$ is Markovian

$|\phi| < 1$

$P(W_t \mid W_{t-1}, \dots, W_0) = P(W_t \mid W_{t-1})$

$$E[W_t] = 0$$

$$\underline{\underline{\gamma_\tau}} = E[\underline{\underline{W_t}} \cdot \underline{\underline{W_{t+\tau}}}] \approx \frac{\sigma^2 \cdot \phi^\tau}{1-\phi^2}$$

$$\boxed{\begin{array}{l} AR(p): \\ \quad W_t = \phi_1 W_{t-1} + \cdots + \phi_p W_{t-p} \end{array}}$$

$w_1 \ldots\ldots w_N$

$$\frac{1}{N-\tau} \sum_{i=1}^{N-\tau} w_i \cdot w_{i+\tau}$$

② Moving average model (MA)

MA(q) $$W_t = \theta_0 \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

$$\left(W_{t-1} = \theta_0 \varepsilon_{t-1} + \theta_1 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-1-q} \right)$$

$$E[W_t] = 0$$

$$Cov(W_t, W_{t+\tau}) = \begin{cases} \sigma^2 \sum_{i=0}^{q-\tau} \theta_i \theta_{i+\tau} & 0 \le \tau \le q \\ 0 & \text{otherwise} \end{cases}$$

③ ARMA(p, q)

$\theta_0 = 1$

$$W_t = \phi_1 W_{t-1} + \phi_2 W_{t-2} \cdots\cdots + \phi_p W_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

$$\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$$

$$\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q$$

① : $\{\text{roots of } \theta(B)\} \cap \{\text{roots of } \phi(B)\} = \cancel{\emptyset}$ empty

② $\phi(B) \neq 0$ for all $|B| \leq 1$: Causality

③ $\theta(B) \neq 0$ for all $|B| \leq 1$. identifiability

$\underline{S_0, S_1, \dots, S_N}$

$\underline{S_1} - S_0, \underline{S_2} - S_1, \dots, S_N - \underline{S_{N-1}}$

S_N	$S_{N-1} \dots S_{N-p}$
S_{N-1}	$S_{N-2} \dots S_{N-p-1}$